Assignment # 2, due February 7

1. p. 36, Problem 1.4

2. p. 37, Problem 1.9

3. The algorithm distributed in class, purported to implement the bisection method, actually has a fatal flaw. (Hint: the loop assumes that the other endpoint of the current best interval is one of the two prior endpoints in the array $Z$, when in fact it might be even farther away.) Rewrite the code in two ways:

(a) Use two arrays, or a matrix of dimensions $\text{numrows} \times 2$, to keep track of both endpoints of the current best interval. The loop should copy one of the endpoints (either to the left or right position in the next row), using the midpoint for the other interval boundary. Your challenge is to implement the logic that correctly decides which of the previous endpoints to keep, and whether it should be on the left or the right.

(b) Keep the array $Z$ as originally designed, but replace $E$ by an array of (signed) interval widths that indicate how far from $Z(\text{iter})$ the other (best current) interval endpoint is. The loop should generate $Z(\text{iter}+1)$ and $E(\text{iter}+1)$ using the appropriate logic for bisection. Check that the plot of $\log |E(\text{iter}+1)|$ versus $\log |E(\text{iter})|$ is much smoother than the jagged curve we saw in class for the flawed implementation.

4. The nonlinear function $f(x) = \int_0^x \frac{2}{\sqrt{t+e^{-t}}} dt$ clearly has a fixed point at $x = 0$. Explain why $f$ must also have a positive fixed point. (Hint: apply the racetrack principle to the identity function and $f$, observing the long-run behavior of $f'(x)$.) Then use simple iteration to approximate the fixed point to 6 significant digits. Feel free to assume that the output of your calculator’s numerical integration routine is accurate enough for our purposes. The syntax is

$$\text{fnInt}(2(t+e^(-t))^{(-1/2)},t,0,x)$$

on TI-83+ and other similar models. In octave you’d use the \texttt{quadv} command, documented at \url{http://www.gnu.org/software/octave/doc/interpreter/Functions-of-One-Variable.html}. 

\[ \text{fnInt}(2(t+e^(-t))^{(-1/2)},t,0,x) \]
5. Write octave code that implements (a) Newton’s method and (b) the secant method for the nonlinear equation \( x = 1 + 4(x + 2)^{\ln(2)/\ln(6)} \). Generate a table similar to Table 1.3 on p. 27 that compares the speed of convergence for the two methods.