Integration Worksheet I

1. We wish to approximate $\int_0^2 e^x \, dx$ by Riemann sums.
   (a) Find the left-hand sum with equal subintervals and with $n = 100$.
   (b) Find the midpoint sum with equal subintervals and with $n = 100$.
   (c) The integral actually equals $e^2 - 1$. Write down the numerical value of $e^2 - 1$, accurate to 8 decimal places. Then determine which of your answers in (a) and (b) is the better approximation. To how many decimal places is it accurate? Why do you suppose it is more accurate?

2. Three groups of calculus students arrive at three different values for $\int_0^{\pi/2} \cos^5 x \, dx$. The claims are:
   Group A says it is 0. Group B says it is $\frac{8}{15}$. Group C says it is 1.6.
   The answer of one group is a good approximation to the value of the integral, and the other two groups gave bad approximations. Without calculating any Riemann sums, tell which two purported values are necessarily in error, and support your answers using complete sentences.

3. It is a fact that we cannot find a simple formula for an antiderivative of $\cos(x^2)$, nor can we find the exact numerical value of $\int_0^1 \cos(x^2) \, dx$. However, one can prove that

   $$1 - \frac{x^4}{2} \leq \cos(x^2) \leq 1 - \frac{x^4}{2} + \frac{x^8}{24} \quad \text{for} \quad 0 \leq x \leq 1.$$

   Use these inequalities and the integral domination theorem to find lower and upper bounds for $\int_0^1 \cos(x^2) \, dx$.

4. The graph of a function $f$ is shown below. Assume that $F'(x) = f(x)$ for all $x$ in $[0, 5]$, and that $F(0) = 2$.
   (a) Find: (i) $F(1)$  (ii) $F(4)$  (iii) $F(5)$
   (b) On each of the unit intervals $[1, 2]$, $[2, 3]$, and $[3, 4]$, determine whether $F$ is increasing or decreasing. Give reasons for your answers. (Hint: How are $F$ and $f$ related?)