1. A storage shed in the shape of a half cylinder (sliced through its axis of symmetry) lies with its flat rectangular side on the ground. The height of the shed is $r$ and the length is $l$.

(a) What is the volume of the shed?

(b) Suppose the shed is filled with sawdust whose density at any point is proportional to the distance of that point from the floor. The constant of proportionality is $k$. Calculate the total mass of sawdust in the shed.

(c) Calculate the center of mass of the sawdust in the shed.

2. A rod of length 2 meters and density $\delta(x) = 3 - e^{-x}$ kilograms per meter is placed on the $x$-axis with its ends at $x = \pm 1$.

(a) Will the center of mass of the rod be on the left or right of the origin? Explain.

(b) Find the coordinate of the center of mass.

3. The following table gives the density $D$ (in g/cm$^3$) of the earth at a depth $x$ km below the earth’s surface. The radius of the earth is about 6370 km. Find an upper and a lower bound for the earth’s mass such that the upper bound is less than twice the lower bound. Explain your reasoning; in particular, what assumptions have you made about the density?

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1000</th>
<th>2000</th>
<th>2900</th>
<th>3000</th>
<th>4000</th>
<th>5000</th>
<th>6000</th>
<th>6370</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>3.3</td>
<td>4.5</td>
<td>5.1</td>
<td>5.6</td>
<td>10.1</td>
<td>11.4</td>
<td>12.6</td>
<td>13.0</td>
<td>13.0</td>
</tr>
</tbody>
</table>

4. We wish to cut out a thin plate of uniform density, using the curve $y = \sec x$ as the upper boundary and the $x$-axis as the lower boundary.

(a) If the left and right boundaries are at $x = \pm \frac{\pi}{4}$, where is the center of mass? (Hint: by symmetry $\bar{x} = 0$, so you only need integration when finding $\bar{y}$.)

(b) If the left and right boundaries are at $x = \pm \frac{\pi}{3}$, where is the center of mass?

(c) Consider a small number $\epsilon$ between 0 and $\frac{\pi}{4}$. If the left and right boundaries are at $x = -\frac{\pi}{2} + \epsilon$ and $x = \frac{\pi}{2} - \epsilon$, where is the center of mass (as a function of $\epsilon$)?

(d) Interpret the results of part (c) physically.
5. Consider a collection of islands, one for each positive integer $n$. The $n$th island has a coastline given by the curves $y = \sin(n\pi x)$, $x = -1$, $x = 1$, and $y = 2$.

(a) Sketch graphs of the second, third, and fifth islands.

(b) Find the length of the coastline for islands 2, 3, and 5. (The arclength integral has to be evaluated numerically, so keep at least 3 decimal places of accuracy.)

(c) Find the area of islands 2, 3, and 5.

(d) Based on your results from parts (b) and (c), make conjectures about the $n$ dependence of coastline length and island area. Confirm your conjectures with an argument based on the relevant integrals for arbitrary $n$. 