Worksheet for Sections 8.3 and 8.4

Name: _____________________________  
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1. In a sentence, describe the difference between integrating by ordinary substitution (as described in Section 5.6) and integrating by trigonometric substitution.

2. For indefinite integrals using trigonometric substitution, it is often necessary to use information from a triangle. In a sentence, describe why one does not need to construct a triangle if the integral is definite.

3. A trigonometric substitution is necessary for only one of the following integrals, while the other two may be handled using ordinary substitution. Evaluate each integral by using an appropriate method, and specify which method you use.

   \[ \int x \sqrt{x^2 - 9} \, dx \]  
   \[ \int \frac{1}{x \sqrt{x^2 - 9}} \, dx \]  
   \[ \int \frac{x}{\sqrt{x^2 - 9}} \, dx \]

4. Where applicable in (a)—(c), perform long division as illustrated in Section 8.3. If long division isn’t needed, say so.

   \[ \frac{x^2}{x^2 - x + 1} \]  
   \[ \frac{x - 1}{x^2 - x + 1} \]  
   \[ \frac{x^4 + 4}{x^2 + 1} \]

5. In the following, factor the numerator and denominator. Then cancel any common factors.

   \[ \frac{x + 3}{x^5 - 81x} \]  
   \[ \frac{x^2 - 2x + 1}{x^3 - 3x^2 + 3x - 1} \]

6. Find the partial fraction decomposition for the following rational functions.

   \[ \frac{6x^2 - 4x + 1}{(x - 2)^3} \]  
   \[ \frac{x^2 + x + 1}{x^4 + 5x^2 + 4} \]  
   \[ \frac{x^3 + 2x^2 + 3x + 4}{x^4 - 16} \]

7. (a) Find \( \int \frac{x^2}{(x^2 - 4)^2} \, dx \). (Hint: \( \frac{x^2}{(x^2 - 4)^2} = \frac{1}{4} \left( \frac{1}{x - 2} + \frac{1}{x + 2} \right)^2 \).) 
   Q. Could the integral be solved by trigonometric substitution? Explain your reasoning.

   (b) Find \( \int \frac{\sqrt{x+4}}{x^3} \, dx \). (Hint: Let \( u = \sqrt{x + 4} \).)